Extra force in Kaluza-Klein gravity theory

W. B. Belayev *

Center for Relativity and Astrophysics,

185 Box , 194358, Sanct-Petersburg, Russia

February 7, 2008

Abstract

In induced matter Kaluza-Klein gravity theory the solution of the dynamics equations for the test particle on null path leads to additional force in four-dimensional space-time. We find such force from five-dimensional geodesic line equations and apply this approach to analysis of the asymmetrically warped space-time.

Keywords: induced matter, geodesics

PACS: 04.50.+h, 04.20.Jb, 04.80.Cc

Five-dimensional space-time theories, including Kaluza-Klein gravity and brane world, analyze extension of 4D space-time. Recently studies of the particle movement in 5D have revealed the consequent departure from 4D geodesic motion by geometric force using some several approaches for its determination [1-7]. In [1] one is found by parametrical differentiating of 4D normalization condition, which is argued by the possibility of the maintenance relation $f^i u_i = 0$, where f^i is component of the extra force (per unit mass) and u^i is 4-velocity. Contrary to this belief principle of the classic general relativity, proposed that free particle moves along his geodesic line, is extended on 5D movement of massive [2-4] and massless [5,6] particles. In induced matter Kaluza-Klein (IM-KK) gravity theory massive particles in 4D have not rest mass in 5D and move on null path [1,5,7].

In this paper through 5D null geodesic motion analysis in IM-KK gravity theory it was found out appearance extra forces in 4D, when the scalar potential depends from coordinates of 4D space-time, and cylinder conditions failed i.e. metric coefficients depend on the fifth coordinate. We analyze example of asymmetrically warped space-time, meaning that the space and time coordinates have different warp factors.

We will consider typical induced matter scenario, when the 5D interval is given by

$$dS^2 = ds^2 - \Phi^2(x^m, y)dy^2, \tag{1}$$

where ds is 4D line element, Φ is scalar potential depended from 4D coordinates x^m and extra space-like coordinate y. The 4D line element is written form

$$ds^2 = g_{ij}(x^m, y)dx^i dx^j, \tag{2}$$

*dscal@ctinet.ru

where g_{ij} is metric tensor.

Since massive in 4D particles move on null path i. e. dS=0, 5D particle dynamics equations are found for null geodesic line just as in 4D by extrimizing [5] function

$$I = \int_a^b d\lambda \{ g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda} - \Phi^2 \frac{dy^2}{d\lambda^2} \} \equiv \int_a^b h d\lambda, \tag{3}$$

where λ is affine parameter along the path of the particle terminated at the points a, b. Null geodesic line equations are given by

$$\frac{d^2 X^A}{d\lambda^2} + \Gamma^A_{BC} \frac{dX^B}{d\lambda} \frac{dX^C}{d\lambda} = 0, \tag{4}$$

where X^A are coordinates of 5D space-time, and Γ^A_{BC} are appropriate defined Christoffel symbols. It should be noticed that if we extremized action with Lagrangian $L \equiv h^{1/2}$ for particle moving on null path in 5D we would obtain division by zero, since this movement assigns $h = 0$. Generally choice of parameter λ is not arbitrary, and turned to differentiation with respect to s in Eq. (4)we obtain

$$\frac{d^2 X^A}{ds^2} + \Gamma^A_{BC} \frac{dX^B}{ds} \frac{dX^C}{ds} = -\omega \frac{dX^A}{ds}, \tag{5}$$

where $\omega = \frac{d^2 X^A}{d\lambda^2} / \left(\frac{ds}{d\lambda} \right)^2$ and interval ds is assumed to be timelike.

The first four components of Eq. (5), corresponded to the motion in 4D spacetime, are transformed to

$$\frac{Du^i}{ds} \equiv \frac{d^2 x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = f^i. \tag{6}$$

Eq.(1) yields for null geodesic:

$$\frac{dy}{ds} = \frac{1}{\Phi}. \tag{7}$$

With this condition for metric (1) with 4D line element (2) fifth force is written as

$$f^i = -\frac{g^{ik}}{\Phi} \left(\frac{\partial \Phi}{\partial x^k} + \frac{\partial g_{kj}}{\partial y} u^j \right) - \omega u^i. \tag{8}$$

The fifth component of Eq. (6) takes the following form:

$$\frac{d^2 y}{ds^2} + \frac{1}{\Phi^2} \left(\frac{\partial g_{ij}}{\partial y} u^i u^j + 2 \frac{\partial \Phi}{\partial x^i} u^i + \frac{1}{\Phi} \frac{\partial \Phi}{\partial y} \right) + \omega \frac{dy}{ds} = 0 \tag{9}$$

By substitution (7) in (9) we obtain

$$\omega = -\frac{1}{\Phi} \left(\frac{\partial g_{ij}}{\partial y} u^i u^j + \frac{\partial \Phi}{\partial x^i} u^i \right). \tag{10}$$

Then equations (8) are rewritten as

$$f^i = -\left(g^{ik} - u^i u^k \right) \frac{1}{\Phi} \left(\frac{\partial \Phi}{\partial x^k} + \frac{\partial g_{kj}}{\partial y} u^j \right). \tag{11}$$

Notice that this formula yields

$$f^i u_i = -\left(\delta^k_l - u^k \right) \frac{1}{\Phi} \left(\frac{\partial \Phi}{\partial x^k} + \frac{\partial g_{kj}}{\partial y} u^j \right) = 0. \tag{12}$$

Let as consider an example, when $\Phi = 1$, and metric (2) is orthogonal and conforms to asymmetrically warped space-time:

$$ds^2 = M(y)\tilde{g}_{00}(x^m)dx^{02} + N(y)\tilde{g}_{ii}(x^m)dx^{i2}, \tag{13}$$

where M, N are functions of extra coordinate. The components of the fifth force (11) are following:

$$\begin{aligned} f^0 &= \left(\frac{M'}{M} - \frac{N'}{N}\right)(M\tilde{g}_{00}u^{02} - 1)u^0, \\ f^i &= \left(\frac{M'}{M} - \frac{N'}{N}\right)M\tilde{g}_{00}u^{02}u^i, \end{aligned} \tag{14}$$

where ($'$) denotes derivative with respect to y, and in second equation velocities u^i conform to the spacelike coordinates.

- This work was supported by MSO.

References

[1] P. S. Wesson, Gen. Rel. Grav. 35(2), 307-317 (2003), gr-qc/0302092.

[2] J. M. Overduin and P.S. Wesson, Phys. Rept. 283, 303 (1997), gr-qc/9805018.

[3] J. Ponce de Leon, Phys. Lett. B523, 311 (2001), gr-qc/0110063.

[4] W. B. Belayev, Spac. Subst. 7, 63 (2001), gr-qc/0110099.

[5] P. S. Wesson and S.S. Seahra, Gen. Rel. Grav. 33, 1731 (2001), gr-qc/0105041.

[6] D. Youm, Mod. Phys. Lett. A16, 2371 (2001), hep-th/0110013.

[7] A. P. Billyard and W. N. Sajko, Gen. Rel. Grav. 33, 1929 (2001), gr-qc/0105074.